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# Study on Real-time co-correction mode in View of Robust Kalman Filtering in Ankang reservoir

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## Abstract

Aiming at the characteristic of Ankang reservoir, the coupling of robustness and kalman filtering was firstly taken in the real-time co-correction model parameters estimation based on the improving Xin'anjiang models. The coupling of robustness and kalman filtering was used to make the dynamic estimation of AR model's error equation parameter, then compared with the outcome of the kalman filtering. The result shows that when hydrological data carries unusual errors, this algorithm could resist the harmful effects and has better tracking ability.

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**Keywords:** real-time co-correction mode; Robust Kalman Filtering; unusual errors; weight function; Ankang reservoir

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## 1. Introduction

Owing to the effect of unnatural factor, such as the measure of telemetry System and hydrographic features, human activity and so on, there are some unusual errors with unknown distribution [1]. In the parameters estimation of the real-time co-correction model, when the measured discharge has the unusual errors, algorithm has better to have capacity of avoiding the influence of the unusual errors and real-time tracking to insure the forecast accuracy[2] [3]. In fact, when the measured discharge has unusual errors, normal kalman filtering isn't able to make effective detection and treatment. Based on this question and the research on this aspect, the coupling of robustness and kalman filtering was firstly taken in the real-time co-correction model parameters estimation, making dynamic estimation of AR model's error equation

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parameters.

In general, reservoir inflow has been got through the observation water level and storage outflow[4]. Obviously, observation water level always has errors, and then converses into measured discharge which also inevitably has unusual errors. And further, due to the affect of human activities, the cascade hydropower development leads that there is no doubt that measured discharge of Downstream Reservoirs' reservoir inflow flood has error unusual errors, which directly lows down the flood forecast accuracy. Take Ankang reservoir as an example, the old flood forecast scheme can't continue to apply because of the influence of upper hydraulic engineerings. Therefore, it's necessary to use the Robust kalman filtering which brings the robust estimation theory into kalman filtering to take insurance for flood forecast accuracy and has an important meaning.

Aim at the characteristic of Ankang reservoir, the coupling of robustness and kalman was taken in the real-time co-correction model parameters estimation based on the improving Xin'anjiang model, making dynamic robust estimation of AR model's error equation parameters.

## 2. Establishment of the real-time co-correction mode in view of robust kalman filtering

In the paper, AR model was chosen as the real-time co-correction model. The coupling of robustness and kalman was taken in the real-time co-correction model parameters estimation based on the improving Xin'anjiang model, making dynamic robust estimation of AR model's error equation parameters. The robust estimation could successfully avoid the influence of unusual errors to achieve the optimum estimation when there are inevitable unusual errors[5].

The improvement of the robust kalman filtering is to choose the proper weight function to replace the observation noise covariance matrix  $R$  based on kalman filtering, which could increase or eliminate the influence of the unusual errors. After make sure the observation noise equivalent covariance matrix, the recursive equation of method of estimation with robust kalman filtering could achieve based on the least squares principle [4].

State predictive value:

$$\hat{X}_{t/t-1} = \phi \hat{X}_{t-1/t-1} \quad (1)$$

Prediction error covariance matrix:

$$P_{t/t-1} = \phi P_{t-1/t-1} \phi^T + \Gamma Q \Gamma^T \quad (2)$$

Gain matrix:

$$G_t = P_{t-1/t-1} \phi^T \left[ \phi P_{t-1/t-1} \phi^T + \hat{R} \right]^{-1} \quad (3)$$

State filtered value :

$$\hat{X}_{t/t} = \hat{X}_{t/t-1} + G_t (Z_t - \phi \hat{X}_{t/t-1}) \quad (4)$$

Filtered error covariance matrix:

$$P_{t/t} = (E - G_t \phi) P_{t-1/t-1} [E - G_t \phi]^T + G_t \hat{R} G_t^T \quad (5)$$

There,  $R$  stands for observation noise covariance matrix,  $\hat{R}$  is the equivalent covariance matrix of  $R$ ,

$P_{t/t}$ ,  $P_{t/t-1}$  in the recursive equation is different from before due to the influence of observation noise equivalent covariance matrix  $\hat{R}$ .

### 2.1. Establishment of state equation and observation equation

How to construct the state equation and observation equation is the key to the research of the Real-time co-correction mode. In this paper, the second order autoregressive equation was chosen as the Real-time co-correction mode, which was coupling with the improving Xin'anjiang model to forecast inflow flood of the Ankang reservoir.

State equation:

$$X_{t+1} = \Phi X_t + \Gamma W_{t+1} \quad (6)$$

$$X_t = \begin{pmatrix} dQ_t \\ dQ_{t-1} \\ 1 \end{pmatrix} \quad \Gamma = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

There,  $X_t$ 、 $X_{t+1}$  respectively stand for state vectors, which was setting as forecasted and measured flow error,  $\Phi$  stands for state transition matrix,  $\Gamma$  stands for noise allocation matrix,  $W_{t+1}$  stands for system noise.

Observation equation:

$$Z_{t+1} = HX_{t+1} + V_t \quad (7)$$

There,  $Z_{t+1}$  stands for flow observed values,  $Z_{t+1} = dQ_{t+1}$ ,  $H$  stands for observed matrix,  $H = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$ ,  $V_t$  stands for system observed noise matrix.

### 2.2 Establishment of robust weight function

Now, there are many ways to choose robust weight function. The common point is to set function of errors as weight of observation value, the difference is the form of the weight function [6]. In the paper, the extensive Huber weight function is chosen to replace observation noise covariance matrix  $R$ .

$$p(V) = \begin{cases} 1 & |V| \leq c \\ \frac{c}{|2V|} & |V| > c \end{cases} \quad (8)$$

$$\sigma^2(k) = \frac{(Z_{t-1} - H_t \hat{X}_{t/t-1})^T (H_t P_{t/t-1} H_t^T) (Z_{t-1} - H_t \hat{X}_{t/t-1})}{n_t} \quad (9)$$

$$V_t = -(E - H_t K_t)(Z_{t-1} - H_t \hat{X}_{t/t-1}) \quad (10)$$

There,  $C$  stands for const, normally  $c = 2\sigma$ ,  $\sigma^2$  stands for unit weight variance,  $n_t$  stands for the number of observation equation.  $Z_{t-1}$  stands for flow observed values,  $Z_{t-1} = dQ_{t-1}$ ,  $V_t$  stands for system

observed noise matrix,  $H$  stands for observed matrix,  $H = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$ ,  $\hat{X}_{t/t-1}$  stands for state vectors, which is got by the last time.  $P_{t/t-1}$  stands for state forecast error covariance matrix, which is got by the last time.  $E$  stands for unit matrix.  $K_t$  stands for gain matrix.

To sum up, the step of making dynamic estimation of AR model's error equation parameters. with the robust kalman filtering is as follow.

Step1: The initial estimation  $\hat{X}_0$  is attained based on  $f$  from the formulation 1 to the formulation 5 with

R, then  $\sigma^2(k)$  and  $V_t$  are got though formulation 9 and formulation 10;

Step2:  $p^{(V)}$  is attained based on formulation 8 with  $\sigma^2(k)$  and  $V_t$ , which are all achieved from Step1;

Step3:  $p^{(V)}$  replaces R as  $\hat{R}$ , then  $\hat{X}_{t+1}$  is attained based on from the formulation 1 to the formulation 5;

Step4:  $\hat{X}_t$  is compared with  $\hat{X}_{t+1}$ . If the difference is lower than the specified error, then stop. Else, go to Step2.

### 3. Results and analysis

Ankang reservoir is a large hydropower complex project which mainly aims at power generation and also can provide service of shipping, flood control and breeding, tourism. Its mean annual discharge is 608 m<sup>3</sup>/s. The river water system is developed. There are 33 rainfall stations and 8 hydrologic station. The old flood forecast scheme can't continue to apply because of the influence of upper hydraulic engineering. Therefore, aim at the characteristic of Ankang reservoir, the coupling of robustness and kalman was taken in the real-time co-correction model parameters estimation based on the improving Xin'anjiang model, making dynamic robust estimation of AR model's error equation parameters.

The river basin, the interval of Shiquan reservoir and Ankang reservoir, was divided into three units. Take the third unit, Lan River, as an example to reveal the realistic significance of the coupling of Xin'anjiang model and Real-time co-correction mode because that the robust effect is more obvious than others. The 30 floods are chosen in the research, but owing to the length of paper, only 7 floods are taken in the paper. The comparison result of kalman filtering and robust kalman filtering revealed in table 1.

Table1. The comparison of kalman filtering and robust kalman filtering

Number of flood	peak discharge(m <sup>3</sup> /s)			peak discharge relative error(%)		deterministic coefficient	
	Measured value	Kalman Filtering	Robust Kalman Filtering	Kalman Filtering	Robust Kalman Filtering	Kalman Filtering	Robust Kalman Filtering
040527	56	69.4	56	23.93	0	0.54	0.92
040829	58.28	69.38	58.27	19.05	-1.71	0.51	0.92
040919	89.64	113.94	89.64	27.11	0.02	0.72	0.92
040930	511	572.1	511.02	11.96	0.37	0.7	0.91
050706	1081	1057.36	1081.2	-2.19	1.81	0.95	0.92
050820	923.2	890.14	923.06	-3.58	-1.49	0.82	0.94

From the table1, it's obvious that: the peak discharge relative errors with robust kalman filtering are obvious smaller than kalman filtering. The former entirely meet the precision requirement of hydrological information and hydrological forecasting (SL250-2000). The deterministic coefficient with Robust Kalman Filtering are obvious smaller than Kalman Filtering.

From the result above, it reveals that: robust kalman filtering is better than kalman filtering in the dynamic robust estimation of AR model's error equation parameters. The reason is that normal kalman filtering isn't able to make effective detection and treatment when the measured discharge has unusual errors.

### 4. Conclusion

The coupling of robustness and kalman was used in the real-time co-correction model parameters

estimation, making dynamic estimation of AR model's error equation parameters. Then, the comparison results of kalman filtering and robust kalman filtering were obtained. The result shows that this algorithm could effectively resist the harmful effects and has better tracking ability than traditional kalman filtering when hydrological data carries unusual errors. It provides a guarantee for stability of system and flood forecast accuracy.

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